HEAT AND MASS TRANSFER IN THE BINARY LAMINARY BOUNDARY LAYER WITH NATURAL CONVECTION

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UDC 532.517.2:536.25

We give approximate analytical solutions for the binary laminar boundary layer; these solutions are based on the Karman–Pohlhausen method and on the method of relative correspondence. The approximate solutions are compared with certain numerical solutions and with the experiment.

Analysis of theoretical references on heat and mass transfer in a binary laminar boundary layer under conditions of natural convection leads to the following conclusions.

The numerical solutions – because of their discrete nature and the rather narrow range of investigated parameters – do not permit generalization nor the derivation of analytical working formulas. In [1], for example, the boundary layer in the case of vapor condensation from a mixture with air is treated exclusively as a purely diffusional phenomenon, with the thermal resistance of the boundary layer neglected; it is only the thermal resistance of the condensate film that is taken into consideration. The calculation is performed for a narrow range of additions of air to the vapor (up to 5% of the weight content of the air) and with three temperature differences. The determination of the heat-transfer coefficient is associated with the extremely difficult method of successive approximations. A solution is derived in [2] for three Grashof numbers and here the transfer of energy as a consequence of the enthalpy difference is neglected, as are the Dufour and Soret effects. The author's assumption that the mass velocity of the wall is equal to zero ($\rho v_w = 0$) led to the fact that the intensity of the heat and mass flow is independent of the concentration difference. Minkowycz and Sparrow [3], examining heat transfer in the condensation of water vapor from a vapor–air mixture, took the thermal diffusion effect into consideration in calculating the heat flow. However, their calculations applied only to one value of the longitudinal coordinate ($x \approx 150$ mm) and to two values for the mass content of the air in the vapor ($m_a = 0.005$ and 0.1).

The injection of hydrogen, helium, water vapor, and carbon dioxide into the air and the injection of a liquid into a solution with $Pr = 10$ and $Sc = 500$ were investigated by a numerical method by Gill et al. [4] for one value of $T_w/T_\infty = 1.1$. Here the authors failed to take into account the Dufour and Soret effects in their expressions for the heat and diffusion flows.

Attempts at analytical solutions with the use of integral boundary-layer equations and the Karman–Pohlhausen method were undertaken in [5-7]. Wilcox made no provision for thermal diffusion and energy transport as a consequence of the enthalpy difference for the components of the mixture in his energy equation, and in the diffusion equation he failed to make any provision for the Soret effect. Moreover, it was assumed in the solution that $Pr = Sc$. Somers solved a system of equations analogous to that of Wilcox; however, $Pr \neq Sc$. In the Baron and Hahn solution provision is made for the diffusive conduction of heat and for the effect of thermal diffusion; the transport of energy resulting from the enthalpy difference is dropped from consideration. However, the errors of this last-cited reference led to imaginary results in the case of suction (condensation) and when $Le \neq 1$.

It is the purpose of this paper to derive the analytical functions to calculate the coefficients of heat and and mass transfer for a binary laminar boundary layer; these relationships should be convenient for practical purposes. In describing the process we sought to account for all forms of energy and mass transport, including the transfer of enthalpy as a consequence of the concentration gradient, the Dufour and Soret

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Institute of Structural Physics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 16, No. 6, pp. 961-971, June, 1969. Original article submitted July 30, 1968.

effects of thermal diffusion. No provision was made in the solution for the work of expansion and viscous dissipation; the properties of the mixture were assumed to be constant, with the exception of the variation in density in the term which provides for the mass forces. The integral equations for the binary boundary layer therefore assume the following form.

The momentum equation

$$\frac{d}{dx}\int_0^h u^2 dy = g\beta_t \int_0^h (t - t_\infty)\, dy + g\beta_m \int_0^h (m_1 - m_{1\infty})\, dy - \nu \left.\frac{\partial u}{\partial y}\right|_w, \tag{1}$$

where β_m is the coefficient of volume expansion, associated with the change in density as a result of the concentration gradient. In particular, for ideal mixtures

$$\beta_m = \left(\frac{M_2}{M_1} - 1\right) \Big/ \left[1 + \left(\frac{M_2}{M_1} - 1\right) m_1\right]. \tag{2}$$

The diffusion equation for the active component is

$$\rho_\infty \frac{d}{dx}\int_0^h u\,(m_1 - m_{1\infty})\, dy = j_{1w} + \rho_w v_w (m_{1w} - m_{1\infty}), \tag{3}$$

where the diffusion flow of the active component, with the consideration of the effect of thermal diffusion (the Soret effect) is defined as

$$j_1 = -\rho D \left[\frac{\partial m_1}{\partial y} + \frac{a_t m_1 (1 - m_1)}{T}\, \frac{\partial t}{\partial y}\right]. \tag{4}$$

The energy equation

$$\frac{d}{dx}\int_0^h u\,(t - t_\infty)\, dy + \frac{c_{p1} - c_{p2}}{\rho_\infty c_p}\int_0^h j_1 \frac{\partial t}{\partial y}\, dy = \frac{q_w}{c_p \rho_\infty} + \bar{\rho} v_w (t_w - t_\infty), \tag{5}$$

where the convection heat flow q_w at the wall is

$$q_w = -\lambda \left.\frac{\partial t}{\partial y}\right|_w + \frac{a_t RMT_w}{427 M_1 M_2}\, j_{1w}. \tag{6}$$

The relationship between the normal velocity v_w at the wall and the diffusion flow of the active component 1, in the case of a semipermeable wall, can be written as

$$v_w = \frac{j_{1w}}{\rho_w (1 - m_{1w})}. \tag{7}$$

The distribution of velocities, temperature, and the mass fraction of component 1 in the boundary layer is taken in the form

$$u = u_1 \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2, \tag{8}$$

$$t - t_\infty = (t_w - t_\infty)\left(1 - \frac{y}{\delta}\right)^2, \tag{9}$$

$$m_1 - m_{1\infty} = (m_{1w} - m_{1\infty})\left(1 - \frac{y}{\delta_m}\right)^2. \tag{10}$$

Integration of (1), (3), and (5), in conjunction with (4), and (6)-(10), is accomplished in the assumption that the dynamic and thermal boundary layers are of equal thickness and this, as will be demonstrated below, introduced no significant error into the expression for the coefficients of heat and mass transfer in the investigated range for Pr = 0.6-10. The solution of the system will then be

$$\frac{\delta}{x}=3{,}93\,(\mathrm{Gr}_x\,\mathrm{Pr})^{-1/4}\left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1w}-m_{1\infty}}{t_w-t_\infty}\,\frac{1}{\xi}\right]^{-1/4}\left[\frac{\mathrm{Pr}}{\eta\,(0.952\eta+\mathrm{Pr})}\right]^{-1/4}, \tag{11}$$

$$\frac{\delta_m}{x}=\frac{3.93}{\xi}\,(\mathrm{Gr}_x\,\mathrm{Pr})^{-1/4}\left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1w}-m_{1\infty}}{t_w-t_\infty}\,\frac{1}{\xi}\right]^{-1/4}\left[\frac{\mathrm{Pr}}{\eta\,(0.952\eta+\mathrm{Pr})}\right]^{-1/4}, \tag{12}$$

$$\mathrm{Re}_{\max}=\frac{u_{\max}x}{\nu}=0.766\,\mathrm{Gr}_x^{1/2}\left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1w}-m_{1\infty}}{t_w-t_\infty}\,\frac{1}{\xi}\right]^{1/2}\frac{\eta}{[\eta\,(0.952\eta+\mathrm{Pr})]^{1/2}}, \tag{13}$$

where

$$\eta=\left\{1+\bar{\rho}\,\mathrm{Le}^{-1}\left(\mathrm{Du}+\frac{1}{1-m_{1w}}\right)[(m_{1w}-m_{1\infty})\,\xi+\mathrm{So}]\right.$$
$$\left.+\,\bar{\rho}\,\mathrm{Le}^{-1}\,\frac{c_{p1}-c_{p2}}{c_p}\left[\xi\left(1-\frac{1}{3}\,\xi\right)(m_{1w}-m_{1\infty})+\frac{2}{3}\,\mathrm{So}\right]\right\}. \tag{14}$$

The ratio of the boundary-layer thicknesses $\xi = \delta/\delta_m$ is determined for $\xi \le 1.0$ ($\delta_m > \delta$) with an accuracy of up to 5% from the quadratic equation

$$\{1+A\,[\mathrm{So}-2.7\,(m_{1w}-m_{1\infty})]\}\,\xi^2-4\left[1+A\,\mathrm{So}-(m_{1w}-m_{1\infty})\,\bar{\rho}\,\mathrm{Le}^{-1}\right.$$
$$\left.\times\left(1.175\,\mathrm{Du}+\frac{1.175}{1-m_{1w}}+\frac{c_{p1}-c_{p2}}{c_p}\right)+\frac{1}{2}\,\bar{\rho}\,\mathrm{Le}^{-1}\,\frac{1-m_{1\infty}}{1-m_{1w}}\right]\xi$$
$$+\,5\left[1+A\,\mathrm{So}-0.4\,\frac{\bar{\rho}\,\mathrm{Le}^{-1}\,\mathrm{So}}{m_{1w}-m_{1\infty}}\,\frac{1-m_{1\infty}}{1-m_{1w}}\right]=0, \tag{15'}$$

where

$$A=\bar{\rho}\,\mathrm{Le}^{-1}\left(\frac{1}{1-m_{1w}}+\mathrm{Du}+\frac{2}{3}\,\frac{c_{p1}-c_{p2}}{c_p}\right).$$

In calculating ξ from (15'), there should be a minus sign in front of the radical.

For $1 \le \xi \le 3.5$ ($\delta > \delta_m$) the value of ξ is found – correct to 2% – from the third-degree equation for ξ

$$1{,}21\,\frac{1}{\xi}-0.195=\bar{\rho}\,\mathrm{Le}^{-1}\,\frac{1-m_{1\infty}}{1-m_{1w}}\left[1+\frac{\mathrm{So}}{m_{1w}-m_{1\infty}}\,\frac{1}{\xi}\right]$$
$$\times\left\{\frac{1}{\xi}+\bar{\rho}\,\mathrm{Le}^{-1}\left[(m_{1w}-m_{1\infty})+\frac{\mathrm{So}}{\xi}\right]\left(\mathrm{Du}+\frac{1}{1-m_{1w}}\right)\right.$$
$$\left.+\,\bar{\rho}\,\mathrm{Le}^{-1}\,\frac{c_{p1}-c_{p2}}{c_p}\left[(m_{1w}-m_{1\infty})\left(1-\frac{\xi}{3}\right)+\frac{2}{3}\,\frac{\mathrm{So}}{\xi}\right]\right\}^{-1}. \tag{15''}$$

Substituting the value of the temperature gradient $\partial t/\partial y|_w = -(2/\delta)(t_w - t_\infty)$ from (9) and the mass content of component 1 (at the wall) $\partial m_1/\partial y|_w = -(2/\delta_m)(m_{1w} - m_{1\infty})$ from (10) into (6) for the convection heat flow at the wall, in conjunction with (11) and (12) we find the dimensionless local heat-transfer coefficient

$$\mathrm{Nu}_x=0.508\,(\mathrm{Gr}_x\,\mathrm{Pr})^{1/4}\left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1w}-m_{1\infty}}{t_w-t_\infty}\,\frac{1}{\xi}\right]^{1/4}$$
$$\times\left[\frac{\mathrm{Pr}}{\eta\,(0.952\eta+\mathrm{Pr})}\right]^{1/4}\{1+\mathrm{Le}^{-1}\,\mathrm{Du}\,[(m_{1w}-m_{1\infty})\,\xi+\mathrm{So}]\}. \tag{16}$$

The local mass-transfer coefficient α_m is determined from the relationship

$$W_{1w}=j_{1w}+\rho_w v_w m_{1w}=\rho_w\alpha_m\,(m_{1w}-m_{1\infty}), \tag{17}$$

where W_{1w} is the total mass flow at the wall, and bearing in mind (7), we find that it is equal to

$$W_{1w}=\frac{j_{1w}}{1-m_{1w}}. \tag{18}$$

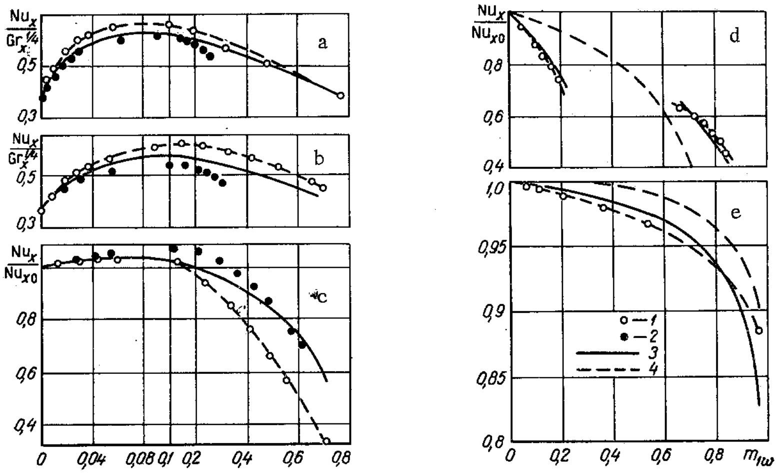

Fig. 1. Injection of hydrogen, helium, water vapor, and carbon dioxide into the air and the injection of liquid into the solution under the conditions of a binary laminar boundary layer in the case of natural convection: 1) numerical solution [4] with consideration of the variability of the physical parameters of the mixture; 2) the same, with constant physical parameters for the mixture, with the exception of the change in density; 3) the approximate solution (16) for Du = 0 and So = 0; 4) solution (30) obtained by the method of relative correspondence; a) H_2–air, $Sc_W/Pr_\infty = 0.35$-2.5; b) He–air, $Sc_W/Pr_\infty = 0.35$-2.5; c) H_2O–air, $Sc_W/Pr_\infty = 0.625$-0.755; d) CO_2–air, $Sc_W/Pr_\infty = 1.2$-1.89; e) solution, $Pr_\infty = 10$, $Sc_\infty = 500$.

Having introduced the values of the temperatures and mass-concentration gradient at the wall into (4), we have

$$W_{1w} = \frac{2\rho_w D}{1-m_{1w}} [(m_{1w}-m_{1\infty})\xi + \mathrm{So}] \frac{1}{\delta}, \tag{19}$$

which with consideration of (11) and (17) yields the local Sherwood number

$$\mathrm{Sh}_x = 0.508\,(\mathrm{Gr}_x \mathrm{Pr})^{1/4} \left[1 + \frac{\beta_m}{\beta_t}\,\frac{m_{1w}-m_{1\infty}}{t_w - t_\infty}\,\frac{1}{\xi}\right] \left[\frac{\mathrm{Pr}}{\eta(0.952\eta + \mathrm{Pr})}\right]^{1/4} \frac{(m_{1w}-m_{1\infty})\xi + \mathrm{So}}{(1-m_{1w})(m_{1w}-m_{1\infty})}. \tag{20}$$

The relationship between Sherwood and the Nusselt numbers is established from comparison of (16) and (20), i.e.,

$$\frac{\mathrm{Sh}_x}{\mathrm{Nu}_x} = \frac{(m_{1w}-m_{1\infty})\xi + \mathrm{So}}{(1-m_{1w})(m_{1w}-m_{1\infty})\{1 + \mathrm{Le}^{-1}\mathrm{Du}\,[(m_{1w}-m_{1\infty})\xi + \mathrm{So}]\}}.$$

With phase conversions or chemical reactions at the wall, the "total" heat flow removed (or brought in) through the wall is expressed as

$$q_w^* = q_w + rW_{1w}, \tag{21}$$

where r is the heat of phase conversion or chemical reaction. The derivation of the heat-transfer coefficient with respect to the "total" heat flow is governed by two circumstances. In engineering heat-transfer calculations the problem in the condensation of a vapor from the vapor–gas mixture is always reduced to the determination of the total heat flow removed by the coolant. In the case of liquid evaporation as a result of the heat introduced by the gas flow, this problem is eliminated, since

$$q_w = -\lambda \left.\frac{\partial t}{\partial y}\right|_w \approx rW_{1w}.$$

When the liquid is evaporated by the heat coming off the solid wall ($T_W > T_\infty$), the "total" heat flow is determined from (21). In addition, the determination of the conditional heat-transfer coefficient through the use of (21) enables us to compare the theoretical solutions with available experimental data on the transfer

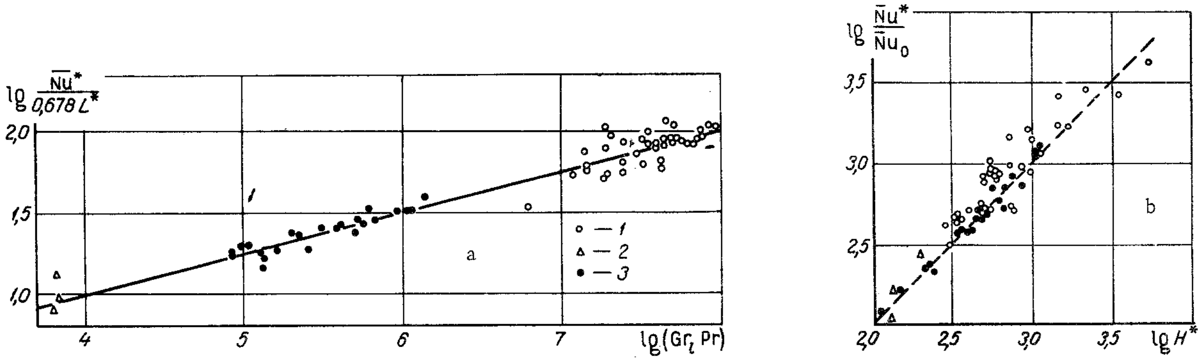

Fig. 2. "Total" heat transfer (with consideration of the heat of phase conversion) on condensation of water vapor from a mixture with air under conditions of laminar natural convection; a) formula (22); b) formula (31); 1) Motulevich experiments [12] on a vertical tube l = 0.5 m; 2) Langen experiments [14] on a horizontal tube d = 30 mm; 3) Othmer experiments [13] on a horizontal tube d = 76.8 mm:

$$L^* = \left[1 + \frac{\beta_m}{\beta_t}\,\frac{m_{1w} - m_{1\infty}}{t_w - t_\infty}\,\frac{1}{\xi}\right]^{1/4} \left[\frac{\Pr}{\eta\,(0.952\eta + \Pr)}\right]^{1/4} \left[1 + \mathrm{Le}^{-1}\,(m_{1w} - m_{1\infty})\,\xi\left(\mathrm{Du} + \frac{K}{1 - m_{1w}}\right)\right];$$

$$H^* = 1 + \left(\frac{\bar{\rho}}{\mathrm{Sc}}\right)^{1/2} \frac{m_{1w} - m_{1\infty}}{1 - m_{1w}} \left(\frac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/2} \left|\frac{\beta_m}{\beta_t}\,\frac{m_{1\infty} - m_{1w}}{t_w - t_\infty}\right|^{1/4}$$

$$\times \left[\frac{0.952 + \Pr}{0.952 + \frac{\mathrm{Sc}}{\bar{\rho}}\,\frac{1 - m_{1w}}{1 - m_{1\infty}}}\right]^{1/4} \left[\frac{\Pr^{1/2}}{\bar{\rho}}\,K - 0.406\left(\frac{M_2}{M_1}\right)^{1/3}\right]$$

of heat in the case of vapor condensation from vapor–gas mixtures in which the heat flow had been measured with consideration of the heats of phase conversion.

Therefore, bearing (21) in mind, we have the local Nusselt number in conjunction with the heats of phase conversion or chemical reaction:

$$\mathrm{Nu}_x^* = 0.508\,(\mathrm{Gr}_x \Pr)^{1/4} \left[1 + \frac{\beta_m}{\beta_t}\,\frac{m_{1w} - m_{1\infty}}{t_w - t_\infty}\,\frac{1}{\xi}\right]^{1/4} \left[\frac{\Pr}{\eta\,(0.952\eta + \Pr)}\right]^{1/4}$$

$$\times \left\{1 + \mathrm{Le}^{-1}\,[(m_{1w} - m_{1\infty})\,\xi + \mathrm{So}]\left(\mathrm{Du} + \frac{K}{1 - m_{1w}}\right)\right\}, \qquad (22)$$

where the conditional heat-transfer coefficient in the value of Nu_x^* is equal to

$$\alpha_x^* = \frac{q_w + rW_{1w}}{t_w - t_\infty}. \qquad (23)$$

The average values of the $\overline{\mathrm{Nu}}$, $\overline{\mathrm{Sh}}$, and $\overline{\mathrm{Nu}}^*$ numbers are found by multiplying their local values by four-thirds.

The method of relative correspondence [8] proposed by Motulevich enables us rather rapidly to derive the output characteristics of the boundary layer. For the base formula in the calculation of heat transfer by the method of relative correspondence, as was done in the calculation of uniform injection and suction in the case of natural convection [9], we use the generalized Gross formula [10] derived for the laminar boundary layer of forced convection:

$$\frac{q_w}{q_{w0}} = 1 - 1.82\left(\frac{M_2}{M_1}\right)^{1/3} \frac{W_{1w}}{\rho_\infty u_1}\left(\frac{u_1 x}{\nu}\right)^{1/2}. \qquad (24)$$

For u_1 we take the characteristic velocity of laminar natural convection on impermeable vertical surfaces [11]:

$$u_1 = 5.17\,\frac{\nu}{x}\,(0.952 + \Pr)^{-1/2}\,\mathrm{Gr}_x^{1/2}. \qquad (25)$$

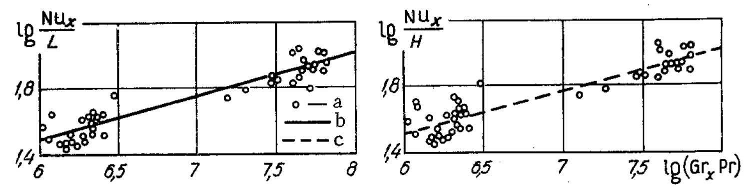

Fig. 3. Local convection heat transfer on condensation of water vapor from humid air under conditions of natural convection; a) experiments [16]; b) formula (16); c) formula (30):

$$L = 0.508\left[1 + \frac{\beta_m}{\beta_t}\,\frac{m_{1w} - m_{1\infty}}{t_w - t_\infty}\,\frac{1}{\xi}\right]^{1/4}\left[\frac{\text{Pr}}{\eta\,(0.952\eta + \text{Pr})}\right]^{1/4} \times \left[1 + \frac{\text{Du}}{\text{Le}}\,(m_{1w} - m_{1\infty})\,\xi\right];$$

$$H = 0.508\left(\frac{\text{Pr}}{0.952 + \text{Pr}}\right)^{1/4}\left\{1 - 0.406\left(\frac{\bar{\rho}}{\text{Sc}}\right)^{1/2}\left(\frac{M_2}{M_1}\right)^{1/3}\frac{m_{1w} - m_{1\infty}}{1 - m_{1w}} \times \left(\frac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/2}\left|\frac{\beta_m}{\beta_t}\,\frac{m_{1w} - m_{1\infty}}{t_w - t_\infty}\right|^{1/4}\left[\frac{0,952 + \text{Pr}}{0,952 + \dfrac{\text{Sc}}{\bar{\rho}}\,\dfrac{1 - m_{1w}}{1 - m_{1\infty}}}\right]^{1/4}\right\}$$

To find the total mass flow of the active component 1 on a semipermeable wall, we solve the problem of mass transfer in the case of natural convection on a vertical surface in approximate terms, with the motion in the laminar boundary layer brought about exclusively by the concentration gradients. In this case the integral equations of momentum and diffusion have the form:

$$\frac{d}{dx}\int_0^h u^2 dy = g\beta_m \int_0^h (m_1 - m_{1\infty})\,dy - \nu\,\frac{\partial u}{\partial y}\bigg|_w, \tag{26}$$

$$\frac{d}{dx}\int_0^h u\,(m_1 - m_{1\infty})\,dy = -\bar{\rho}D\,\frac{\partial m_1}{\partial y}\bigg|_w + \bar{\rho}v_w\,(m_{1w} - m_{1\infty}). \tag{27}$$

The distribution of the velocities and the mass fraction of component 1 are taken, respectively, from (8) and (10). With consideration of (7), the solution of (26) and (27) will then be:

$$\delta_m = 3.93\left(\frac{\bar{\rho}}{\text{Sc}}\right)^{1/2}\left(\frac{1 - m_{1\infty}}{1 - m_{1w}}\right)^{1/2}\left[0.952 + \frac{\text{Sc}}{\bar{\rho}}\,\frac{1 - m_{1w}}{1 - m_{1\infty}}\right]^{1/4}\left[\frac{g\beta_m|m_{1w} - m_{1\infty}|}{\nu^2}\right]^{-1/4} x^{1/4}. \tag{28}$$

Bearing in mind that the diffusion flow at the wall is given by $j_{1W} = -\rho_W D(\partial m_1/\partial y)|_W$, and that the gradient of concentration for component 1 at the wall from (10) is equal to $(\partial m_1/\partial y)|_W = -2(m_{1W} - m_{1\infty})(1/\delta_m)$, from (18), with the consideration of (28), we obtain:

$$W_{1w} = 0.508\,\rho_w\,\frac{D}{x}\left(\frac{\text{Sc}}{\bar{\rho}}\right)^{1/2}\frac{m_{1w} - m_{1\infty}}{1 - m_{1w}}\left(\frac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/2}\frac{\text{Gr}_{xm}^{1/4}}{\left(0,952 + \dfrac{\text{Sc}}{\bar{\rho}}\,\dfrac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/4}}, \tag{29}$$

where $\text{Gr}_{xm} = g\beta_m|m_{1W} - m_{1\infty}|x^3/\nu^2$ is the concentration Grashof number.

Substitution of (25) and (29) into (24) finally yields

$$\frac{\text{Nu}_x}{\text{Nu}_{x0}} = 1 - 0.406\left(\frac{\bar{\rho}}{\text{Sc}}\right)^{1/2}\left(\frac{M_2}{M_1}\right)^{1/3}\frac{m_{1w} - m_{1\infty}}{1 - m_{1w}}\left(\frac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/2} \times \left|\frac{\beta_m}{\beta_t}\,\frac{m_{1w} - m_{1\infty}}{t_w - t_\infty}\right|^{1/4}\left(\frac{0.952 + \text{Pr}}{0,952 + \dfrac{\text{Sc}}{\bar{\rho}}\,\dfrac{1 - m_{1\infty}}{1 - m_{1w}}}\right)^{1/4}, \tag{30}$$

where Nu_{x0} is the value of the local Nusselt number on the impermeable surface.

With phase conversions or chemical reactions at the wall, the dimensionless conditional coefficient of heat transfer is calculated from (23):

$$\frac{\mathrm{Nu}_x^*}{\mathrm{Nu}_{x0}} = 1 + \left(\frac{\bar{\rho}}{\mathrm{Sc}}\right)^{1/2} \frac{m_{1w} - m_{1\infty}}{1 - m_{1w}} \left(\frac{1 - m_{1w}}{1 - m_{1\infty}}\right)^{1/2} \left| \frac{\beta_m}{\beta_t} \, \frac{m_{1w} - m_{1\infty}}{t_w - t_\infty} \right|^{1/4}$$

$$\times \left[\frac{0.952 + \mathrm{Pr}}{0.952 + \frac{\mathrm{Sc}}{\bar{\rho}} \, \frac{1 - m_{1w}}{1 - m_{1\infty}}}\right]^{1/4} \left[\frac{\mathrm{Pr}^{1/2}}{\bar{\rho}} K - 0.406 \left(\frac{M_2}{M_1}\right)^{1/3}\right]. \tag{31}$$

Figure 1 shows a comparison of the approximate solution with the numerical calculation of Gill et al. [4], where the Dufour and Soret effects were neglected. The numerical calculation was performed for two cases: with consideration of the variability of the physical parameters of the mixture (ρ, μ, λ, c_p, and Sc = var) as functions of the temperature and the concentrations of the injected components; the parameters of the mixture were assumed to be constant, with the exception of the change in the density of the mixture, which was a function of the temperature and concentration. On injection of hydrogen into the air, the physical parameters of the mixture varied within the following limits: $\bar{\rho} = 0.08$–0.9; $\bar{\mu} = 0.532$–1.0; $\bar{\lambda} = 1.0$–6.4; $\bar{c}_p = 1$–11; $\overline{\mathrm{Sc}} = 1.0$–$6.1$; Pr = 0.72. On injection of helium into the air $\bar{\rho} = 0.17$–0.9; $\bar{\mu} = 1.0$–1.1; $\bar{\lambda} = 1.0$–4.75; $\bar{c}_p = 1.0$–3.9; $\overline{\mathrm{Sc}} = 1.0$–$5.95$; Pr = 0.6. On injection of water vapor $\bar{\rho} = 0.63$–0.9; $\bar{\mu} = 0.67$–1.0; $\bar{\lambda} = 0.87$–1.0; $\bar{c}_p = 1.0$–1.7; $\overline{\mathrm{Sc}} = 0.96$–$1.0$; Pr = 0.9. On injection of carbon dioxide $\bar{\rho} = 0.9$–1.28; $\bar{\mu} = 0.87$–1.0; $\bar{\lambda} = 0.78$–1.0; $\bar{c}_p = 0.94$–1.0; $\overline{\mathrm{Sc}} = 0.62$–$1.4$; Pr = 1.0.

On injection of hydrogen and helium the approximate solution of (16) yields somewhat understated results (the maximum divergence does not exceed 9%) in comparison with the numerical solution performed with consideration of the variability of the physical parameters of the mixture. On injection of CO_2 and with injection of liquid the results are virtually coincident. On injection of water vapor to a wall concentration of $m_{1w} = 0.1$ the solutions coincide; for large concentrations solution (16) leads to overstated data (the maximum divergence reaches 30% with a concentration $m_{1w} = 0.6$).

Even better agreement between the approximate solution (16) and the numerical results was found if we were able to assume the physical parameters of the mixture constant in the numerical results (with the exception of the change in the density of the mixture with variation in temperature and concentration).

The numerical calculations [4] with a CO_2 concentration of 0.6 at the wall are debatable, since at these concentrations and at a temperature ratio of $T_w/T_\infty = 1.1$ the solution leads to imaginary values of the heat-transfer coefficient, which is probably brought about by the inaccuracy of formula (2) in so far as it is used to calculate the concentration coefficient of the volume expansion for large mass concentrations of the injected gas, in the case in which that gas is heavier than the main gas. In [4], with $(m_{CO_2})_w > 0.6$, the lift forces which are produced by the concentration gradient are therefore neglected in the calculations.

Figure 2 shows a comparison of the approximate solutions (22) and (31) for the average heat-transfer coefficients calculated with consideration of the heat of phase conversion (formula (21)) with the heat-transfer experiments in the case of water-vapor condensation from a mixture with air on vertical and horizontal tubes [12-14]. The height of the vertical tubes was used as the decisive dimension, and in the case of horizontal tubes it was the diameter. The physical parameters of the mixture were referred to the temperature and concentration at "infinity." The thermal conductivity, the viscosity, and the thermal-diffusion constant of the mixture were evaluated on the basis of the modified Buckingham potential [15]. The partial pressure of the water vapor at the wall was calculated from the average temperature of the solid wall.

The limits of generalization for the experimental data were the following: the mass content of the air in the vapor is $m_a^\infty = 0.015$–0.18, for the Kutateladze number we have K = 30-1600; for the pressure of the mixture we have $p_m = 0.5$–1.4 atm abs.; the Rayleigh number $\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = 6 \cdot 10^3$–$10^8$, the Lewis number Le = 0.56-0.9, and the Dufour number Du = 0.15-6.6.

Comparison of the experiment on local convection heat transfer in the case of water-vapor condensation from moist air [16] with the approximate solutions (16) and (30) is shown in Fig. 3. In the experiments of [16] the volume content of the vapor in the air varied from 1.5 to 16%.

As we can see from Figs. 1-3, the approximate solutions yield good agreement both with the numerical calculations and the experiment over a wide range of decisive process parameters.

NOTATION

x and y	are the longitudinal and transverse coordinates;
u and v	are the longitudinal and transverse velocity components;
T	is the absolute temperature;
t	is the temperature;
m_i	is the mass fraction of the i-th component;
β_t and β_m	are the thermal and concentration coefficients of volume expansion;
ρ	is the density;
c_p	is the isobaric heat capacity;
μ	is the coefficient of viscosity;
D	is the coefficient of diffusion;
a	is the thermal diffusivity;
a_t	is the thermal-diffusion constant;
λ	is the coefficient of thermal conductivity;
M	is the molecular weight;
R	is the gas constant;
r	is the heat of phase conversion;
q	is the convection heat flow;
q^*	is the heat flow, with consideration of the phase conversions or chemical reactions;
j_i	is the diffusion flow of the i-th component;
δ	is the thickness of the thermal boundary layer;
δ_m	is the thickness of the diffusion boundary layer;
$\xi = \delta/\delta_m$;	
W_i	is the total mass flow of the i-th component;
α	is the coefficient of convection heat transfer;
α^*	is the conditional coefficient of heat transfer with consideration of the heat of phase conversion;
α_m	is the coefficient of mass transfer;
Nu	is the convection Nusselt number;
Nu*	is the Nusselt number derived on the basis of α^*;
$\mathrm{Sc} = \nu/D$	is the Schmidt number;
$\mathrm{Le} = D/a$	is the Lewis number;
$\mathrm{Du} = (a_t R M^2/427\, M_1 M_2 c_p) \cdot (T_w/(T_w - T_\infty))$	is the Dufour number;
$\mathrm{So} = a_t m_{1w}(1 - m_{1w}) \cdot ((T_w - T_\infty)/T_w)$	is the Soret number;
$\mathrm{Sh} = \alpha_m x/D$	is the Sherwood number;
$K = r/c_p(t_w - t_\infty)$	is the Kutateladze number;
Pr	is the Prandtl number;
$\bar{\rho} = \rho_w/\rho_\infty$;	
$\bar{\mu} = \mu_w/\mu_\infty$;	
$\bar{\lambda} = \lambda_w/\lambda_\infty$;	
$\underline{c}_p = c_{pw}/c_{p\infty}$;	
$S_c = Sc_w/Sc_\infty$.	

Subscripts

w denotes the wall;
∞ denotes conditions outside the boundary layer;
0 denotes a value at the impermeable wall;
1 denotes the active component of the binary mixture;
2 denotes the inert component of the binary mixture;
x denotes a local value;
L denotes a value taken along the length L.

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